Available online at www.sciencedirect.com
ScienceDirect
International Journal of Approximate Reasoning
47 (2008) 219–232
INTERNATIONAL JOURNAL OF APPROXIMATE REASONING
www.elsevier.com/locate/ijar

Reasoning about topological relations between regions with broad boundaries

Shihong Du [a,*], Qimin Qin [a], Qiao Wang [b], Haijian Ma [a]

[a] *Institute of Remote Sensing and GIS, Peking University, Beijing 100871, China*
[b] *School of Geographic Science, Nanjing Normal University, Nanjing 210097, China*

Received 3 July 2006; received in revised form 28 March 2007; accepted 18 May 2007
Available online 5 June 2007

Abstract

Uncertain regions can be represented as having broad boundaries (BBRs) and their topological relations can be modeled by the extended 9-intersection. In order to satisfy the need for querying, managing, and processing BBRs, this study presents a 4-tuple representation of topological relations between BBRs, and a method in which the relations between simple regions with broad boundaries (SBBRs) are used to infer new topological information. The 4-tuple representation can distinguish the same topological relations as identified by the extended 9-intersection. Since the 4-tuple uses combinations of the basic topological relations between crisp regions to describe the relations between uncertain regions, the reasoning of topological relations between SBBRs can be obtained by combining the results of those between crisp regions. The reasoning mechanism can be used in several applications, such as to evaluate the consistency of topological relations between uncertain regions in multi-resolution spatial databases and to assess the consistency of a complete or incomplete symbolic description of a spatial scene.

Keywords: Geographical information system; 4-tuple representation; Regions with broad boundaries; Reasoning of topological relations

1. Introduction

Geographical information systems (GISs) provide users with the ability to manage, query, analyze, and display spatial data, as well as support decision-making in many geographical applications. Traditional GISs only deal with geographical phenomena modeled by crisp points, lines, and regions that are clearly defined or have crisp boundaries. This is unfortunately not the case for many phenomena, some of which are too difficult or too expensive to be determined accurately while others change so fast that it is impossible to measure them exactly. Nonetheless, a model for representing the uncertainty of geospatial data is needed.

There are two kinds of geographical phenomena in the real world. The boundaries of the first kind can be defined clearly or determined accurately, such as the location of a house, road, etc., which allows these objects

* Corresponding author. Tel.: +86 010 62764430; fax: +86 010 62751691.
E-mail addresses: dshgis@hotmail.com (S. Du), qmqin@pku.edu.cn (Q. Qin), wangqiao@zhb.gov.cn (Q. Wang).

0888-613X/$ - see front matter
doi:10.1016/j.ijar.2007.05.002

to be modeled with crisp boundaries. The boundaries of the second kind are uncertain and cannot be defined clearly or determined accurately, such as soil type and suburbs. Geographical phenomena with uncertain boundaries can be modeled by regions with broad boundaries (BBRs). A region with broad boundary, A, can be thought of as a pair, (a_1, a_2), of closed disc-like crisp regions in the plane such that $a_1 \subseteq a_2$. The broad boundary is defined by $a_2 - a_1^\circ$. BBRs use the broad boundary to represent the spatial extent of the uncertain boundary, while overlooking the degree of uncertain membership of every point in the broad boundary. Therefore, they can model geospatial phenomena characterized by various types of uncertainty [6,25] including: (1) incomplete representations of geospatial data; (2) inconsistent representations of the same objects in the integration of multi-resource geospatial data; (3) dynamic geospatial phenomena varying over time; (4) incomplete observations of geospatial phenomena; (5) inherent fuzziness of geospatial phenomena.

The formalization and reasoning of topological relations are essential research topics with widespread applications in managing, querying, and analyzing geospatial data, such as spatial data query [18], spatial data mining [7], image retrieval based on content [22,26], equivalence and similarity of spatial scenes [20], and consistent maintenance of multi-resolution spatial databases [3,14,15,17,21,24]. Most of these applications, however, can only deal with topological relations between crisp objects, since topological relations between BBRs have not been developed to the extent that they fulfill the needs of most applications, such as consistency detection of topological relations and the topological equivalence and similarity of BBRs in multi-resolution spatial databases, etc.

To formalize topological relations between crisp objects, the 4-intersection model [11], the 9-intersection model [12], the Voronoi-based 9-intsection model [9], the general intersection model [1,2], and the calculus-based method [4] have been proposed. Clementini and Di Felice [5] used the extended 9-intersection to describe topological relations between BBRs by replacing the crisp boundary in the 9-intersection with the broad boundary; they then investigated the *closest-topological-relation* graph [6] and three-level operators to support the query of uncertain regions [8].

Given two topological relations, between A and B and B and C, the goal of the reasoning of topological relations is to use them to infer all possible relations between objects A and C. Egenhofer [16] presented a method to compute the composition of two binary topological relations between two regions, with the results represented as an 8×8 table in which any composition between two topological relations can be found. This method only applies to the reasoning of topological relations between two crisp regions and is based on the 9-intersection approach. Abdelmoty and El-Geresy [1] pointed out that existing methods of spatial reasoning only focused on spatial relations between objects of similar type having the same dimensions and simple shapes. To overcome this limitation, they extended the 9-intersection to a general intersection model for formalizing topological relations between objects with arbitrary type and complex shape [2], and then proposed a general method for computing composition tables [1]. Egenhofer and Al-Taha [13] investigated the topological changes caused by the translation, rotation, reduction, and expansion of crisp regions. First, they defined the topology distance between two topological relations; second, a *closest-topological-relation* graph was constructed in terms of the topological distance; and third, the topological changes introduced by translation, rotation, and scaling were interpreted as distinct paths in the graph. Most of the models developed thus far have focused on the reasoning of the relations between crisp objects. To our knowledge, however, the reasoning of topological relations between BBRs remains unresolved.

The present work focuses on computation methods for the reasoning of topological relations between simple regions with broad boundaries (SBBRs). In the approach described herein, the basic relations between inner–inner, inner–outer, outer–inner, and outer–outer regions of two SBBRs are defined based on sets of points. A 4-tuple representation of topological relations between SBBRs is then generated by combining the four relations between inner and outer regions. Finally, the reasoning of topological relations between SBBRs is computed by combining the results obtained from the reasoning of the four topological relations between inner and outer regions. In addition, the 4-tuple representation, based on a reduced set of topological relations between inner and outer regions, is shown to have the same ability as the extended 9-intersection to discern topological relations between SBBRs, while the 4-tuple approach is helpful to compute the reasoning. The reasoning of topological relations between BBRs can be applied to the following situations:

- Determining whether certain objects in the physical world satisfy the topological relations contained in a query sentence before it is submitted for processing.
- Detecting the inconsistency of topological relations between BBRs in multi-resolution spatial databases.
- Evaluating the topological equivalence and similarity of BBRs in multi-resolution spatial databases.
- Assessing the consistency of a complete or incomplete symbolic description of a spatial scene encompassing BBRs.
- Modeling and deriving topological relations in an uncertain environment for way-finding, robot route planning, and other fields of cognitive science [19].

Related work on topological relations between crisp regions, and between BBRs is discussed in Section 2. Section 3 presents the 4-tuple representation of topological relations between BBRs. The method for deriving topological relations between SBBRs is described in Section 4. Section 5 discusses the reasoning of hierarchical topological relations, which are clusters of the basic topological relations between SBBRs. Section 6 compares the 4-tuple representation and the RCC-5 model. Future work is suggested in Section 7.

2. Qualitative topological relations between regions

2.1. The 9-intersection model for simple and crisp regions

The topological relations between simple and crisp regions can be determined by the 9-intersection model [12], which divides a plane into three parts: interior (A°), boundary (∂A), and exterior (A^{-}). The topological relations between two crisp regions A and B, $T(A,B)$, can be determined by the nine intersections between the three parts (Eq. (1)). Since the intersection of two sets can be either 0 or 1, the nine intersections can determine 512 topological relations in theory, but only eight basic ones, named *disjoint*, *meet*, *overlap*, *contain*, *equal*, *coveredBy*, *inside*, and *cover*, are possible for 2-dimensional simple regions in the physical world (Fig. 1).

$$T(A,B) = \begin{bmatrix} A^{\circ}\cap B^{\circ} & A^{\circ}\cap \partial B & A^{\circ}\cap B^{-} \\ \partial A\cap B^{\circ} & \partial A\cap \partial B & \partial A\cap B^{-} \\ A^{-}\cap B^{\circ} & A^{-}\cap \partial B & A^{-}\cap B^{-} \end{bmatrix} \tag{1}$$

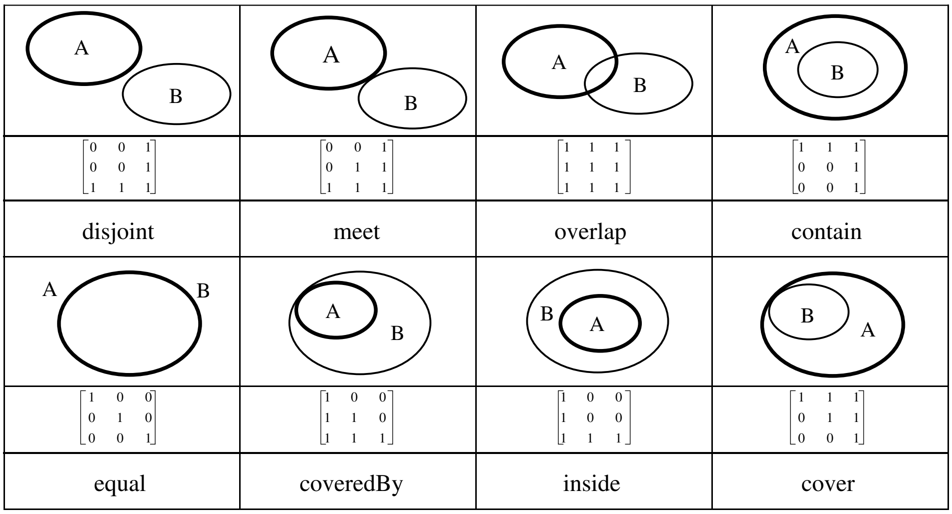

Fig. 1. The eight basic topological relations between two crisp regions [12].

2.2. The extended 9-intersection for SBBRs

A region with broad boundary A, composed of a pair of closed disc-like regions a_1 and a_2 in the plane such that $a_1 \subseteq a_2$, divides a plane into three disjoint subsets:

- The interior of A, denoted by A°, defined as the interior of a_1°.
- The broad boundary, denoted by A^Δ, defined as $a_2 - a_1^\circ$.
- The exterior, denoted by A^-, which is the set-theoretic complement of a_2.

Clementini and Di Felice [5,6] extended the 9-intersection for crisp regions by replacing the crisp boundary with the broad boundary to formalize topological relations between SBBRs (Eq. (2)). The extended 9-intersection, which depends on nine intersections between the interior, broad boundary, and exterior, can

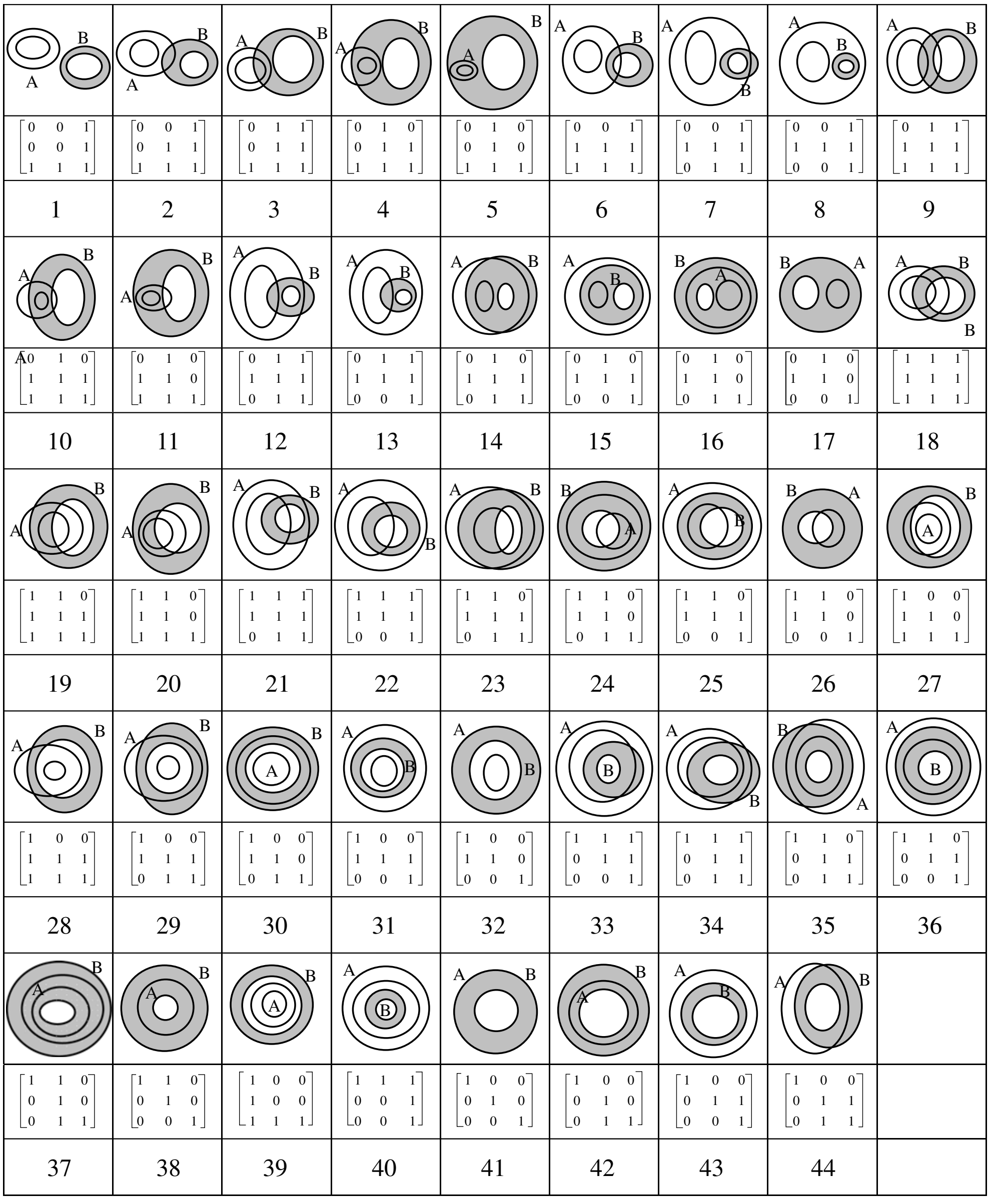

Fig. 2. The 44 realizable topological relations between SBBRs [8].

determine 44 realizable topological relations between SBBRs (Fig. 2). These topological relations, in terms of the similarities of their geometric properties, were grouped into a three-level hierarchy, i.e., bottom, intermediate, and top, to support queries regarding BBRs [8].

$$T(A,B) = \begin{bmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap B^{\Delta} & A^{\circ} \cap B^{-} \\ A^{\Delta} \cap B^{\circ} & A^{\Delta} \cap B^{\Delta} & A^{\Delta} \cap B^{-} \\ A^{-} \cap B^{\circ} & A^{-} \cap B^{\Delta} & A^{-} \cap B^{-} \end{bmatrix} \quad (2)$$

3. The 4-tuple representation of topological relations between SBBRs

The extended 9-intersection considers SBBRs as a whole and uses the nine intersections among three subsets of two SBBRs to describe the topological relations. But how do the 44 relations determined by the extended 9-intersection is related to the four relations: between a_1 and b_1, between a_1 and b_2, between a_2 and b_1, and between a_2 and b_2? Our 4-tuple method represents topological relations between SBBRs as the combination of these four relations. Compared to the 9-intersection model, the 4-tuple representation can make finer distinctions, as it is more expressive than the extended 9-intersection. However, it is not necessary to distinguish more cases, because the goal is to find a model equivalent to the extended 9-intersection and expressed with crisp simple regions. Thus, our model can be used to establish the full 44×44 composition table for two SBBRs by reapplying the composition tables for simple regions. To perform this task, the four relations in the 4-tuple representation are modeled by a reduced set of topological relations, not by the eight topological relations. Specifically, the ability of the 4-tuple representation to describe topological relations is the same as that of the extended 9-intersection; in other words, the two models determine the same number and types of topological relations. The 4-tuple, however, can be applied to the reasoning of topological relations between BBRs, because it uses the composition of topological relations between crisp regions to determine those between uncertain regions.

3.1. The 4-tuple representation

An arbitrary SBBR, A, is composed of two crisp regions: an inner region, a_1, and an outer region, a_2. Each crisp region divides the plane into three parts: interior, crisp boundary, and exterior. Further, the topological relations between crisp regions can be determined by the 9-intersection. Therefore, the topological relations between two SBBRs, A and B, can be treated as the combination of four such relations between a_1 and b_1, a_1 and b_2, a_2 and b_1, and a_2 and b_2. Each topological relation between two crisp regions can be any one of the eight basic topological relations determined by the 9-intersection.

Since the broad boundary is defined as $a_2 - a_1^{\circ}$ in the extended 9-intersection, many of the topological relations between SBBRs, which are different viewed from the four topological relations between inner and outer regions, are the same topological relations in terms of the extended 9-intersection. In Fig. 3, the three topological relations corresponds to the same relations in terms of the extended 9-intersection, whereas they are different in terms of the four relations. For example, the topological relations between a_1 and b_1, and a_2 and b_1 are always *disjoint* in Fig. 3; $T(a_1,b_2) =$ *disjoint*, $T(a_2,b_2) =$ *overlap* in Fig. 3a; $T(a_1,b_2) =$ *meet*, $T(a_2,b_2) =$ *overlap* in Fig. 3b; $T(a_1,b_2) =$ *disjoint*, $T(a_2,b_2) =$ *meet* in Fig. 3c. This indicates that if the eight basic topological

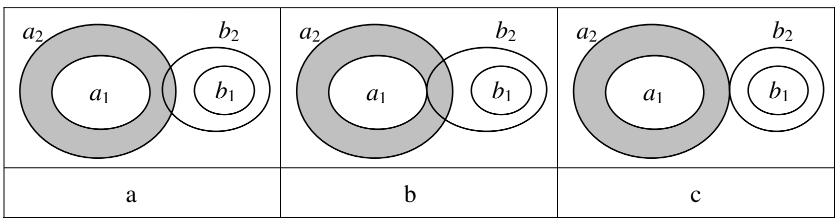

Fig. 3. The geometry examples of the same topological relation between BBRs.

relations are used to represent the four relations between a_1 and b_1, a_1 and b_2, a_2 and b_1, and a_2 and b_2, then the 4-tuple is more expressive than the extended 9-intersection. Therefore, to find a 4-tuple equivalent to the extended 9-intersection, it is sufficient to adopt a reduced set of topological relations to define the four relations.

To find an equivalent model with the extended 9-intersection, and then relate the 44 relations to the 4-tuples, the definitions of the interior, the broad boundary and the exterior in the extended 9-intersection should be considered in the formalization of the four relations. In the extended 9-intersection, the interior and the exterior are open sets, and the broad boundary is a closed set. Therefore, to keep the 4-tuple consistent with the extended 9-intersection, the four relations between a_1° and b_1°, a_1° and b_2, a_2 and b_1°, and a_2 and b_2 are used to represent the topological relations between two SBBRs. The four relations can be determined by checking conditions between non-empty sets of points, as described in Table 1.

In Table 1, symbol a and b represent two non-empty sets of points. If a and b are used to represent the four pairs of point sets, a_1° and b_1°, a_1° and b_2, a_2 and b_1°, and a_2 and b_2, respectively, then the basic relations of the four pairs of point sets can be determined by checking conditions. For example, the relation between a_1° and b_1°, denoted by $T(a_1,b_1)$, or between a_2 and b_2, denoted by $T(a_2,b_2)$, is one of the five relations *disjoint′*, *overlap′*, *contain′*, *inside′*, and *equal′*; while the relation between a_1° and b_2, denoted by $T(a_1,b_2)$, or between a_2 and b_1°, denoted by $T(a_2,b_1)$, is one of the four relations *disjoint′*, *overlap′*, *contain′*, and *inside′*, because a closed set (a_2 or b_2) can never be equal to an open set (a_1° or b_1°). These five or four basic relations are mutually exclusive and cover all possible relations between two non-empty sets of points.

In order to compute the composition of topological relations between SBBRs and relate the reduced set of five relations with the eight ones of the 9-intesection, it is necessary to find the correspondences among the four relations and the topological relations determined by the 9-intersection. By an analysis of the 9-intersection model (Fig. 2) and the definitions of the four relations in the 4-tuple (Table 1), the correspondences are obtained (Table 2).

Definition 1. Assuming that a SBBR, A, consists of an inner region, a_1, and an outer region, a_2, then the topological relations between SBBRs A and B can be represented as a 4-tuple $\langle T(a_1,b_1), T(a_1,b_2), T(a_2,b_1), T(a_2,b_2)\rangle$.

According to Definition 1, based on the four relations: $T(a_1,b_1)$, $T(a_1,b_2)$, $T(a_2,b_1)$, and $T(a_2,b_2)$, a topological relation between SBBRs can be represented as a 4-tuple. For example, the first topological relation in Fig. 2 can be represented as $\langle$*disjoint′*,*disjoint′*,*disjoint′*,*disjoint′*$\rangle$, the second as $\langle$*disjoint′*,*disjoint′*,*disjoint′*,*overlap′*$\rangle$, the fifth as $\langle$*disjoint′*,*inside′*,*disjoint′*,*inside′*$\rangle$, and the ninth as $\langle$*disjoint′*,*overlap′*,*overlap′*,*overlap′*$\rangle$. For the 44 realized topological relations, the corresponding 4-tuples are listed in Table 3, where *d*, *o*, *e*, *i*, and *c* represent, respectively, the basic topological relations, *disjoint′*, *overlap′*, *equal′*, *inside′*, and *contain′*.

Table 1
Basic relations between two non-empty sets of points

Relation	Condition
a is *disjoint′* from b	$a \cap b = \emptyset$
a *overlaps′* b	$a \cap b \neq \emptyset$ and $a \cap b \neq a$ and $a \cap b \neq b$
a *contains′* b	$a \cap b \neq \emptyset$ and $a \cap b \neq a$ and $a \cap b = b$
a is *inside′* b	$a \cap b \neq \emptyset$ and $a \cap b = a$ and $a \cap b \neq b$
a *equals′* b	$a \cap b \neq \emptyset$ and $a \cap b = a$ and $a \cap b = b$

Table 2
Relationships between the eight basic relations and the five basic ones

	disjoint′	*overlap′*	*contain′*	*inside′*	*equal′*
$T(a_1,b_1)$	*disjoint*, *meet*	*overlap*	*contain*, *cover*	*inside*, *coveredBy*	*equal*
$T(a_1,b_2)$	*disjoint*, *meet*	*overlap*, *cover*	*contain*	*inside*, *coveredBy*	*
$T(a_2,b_1)$	*disjoint*, *meet*	*overlap*, *coveredBy*	*contain*, *cover*	*inside*	*
$T(a_2,b_2)$	*disjoint*	*overlap*, *meet*	*contain*, *cover*	*inside*, *coveredBy*	*equal*

Table 3
The one-to-one mapping between the 4-tuple and extended 9-intersection

E9I	4-tuple	E9I	4-tuple	E9I	4-tuple	E9I	4-tuple
1	$\langle d,d,d,d\rangle$	2	$\langle d,d,d,o\rangle$	3	$\langle d,o,d,o\rangle$	4	$\langle d,i,d,o\rangle$
5	$\langle d,i,d,i\rangle$	6	$\langle d,d,o,o\rangle$	7	$\langle d,d,c,o\rangle$	8	$\langle d,d,c,c\rangle$
9	$\langle d,o,o,o\rangle$	10	$\langle d,i,o,o\rangle$	11	$\langle d,i,o,i\rangle$	12	$\langle d,o,c,o\rangle$
13	$\langle d,o,c,c\rangle$	14	$\langle d,i,c,o\rangle$	15	$\langle d,i,c,c\rangle$	16	$\langle d,i,c,i\rangle$
17	$\langle d,i,c,e\rangle$	18	$\langle o,o,o,o\rangle$	19	$\langle o,i,o,o\rangle$	20	$\langle o,i,o,i\rangle$
21	$\langle o,o,c,o\rangle$	22	$\langle o,o,c,c\rangle$	23	$\langle o,i,c,o\rangle$	24	$\langle o,i,c,i\rangle$
25	$\langle o,i,c,c\rangle$	26	$\langle o,i,c,e\rangle$	27	$\langle i,i,o,i\rangle$	28	$\langle i,i,o,o\rangle$
29	$\langle i,i,c,o\rangle$	30	$\langle i,i,c,i\rangle$	31	$\langle i,i,c,c\rangle$	32	$\langle i,i,c,e\rangle$
33	$\langle c,o,c,c\rangle$	34	$\langle c,o,c,o\rangle$	35	$\langle c,i,c,o\rangle$	36	$\langle c,i,c,c\rangle$
37	$\langle c,i,c,i\rangle$	38	$\langle c,i,c,e\rangle$	39	$\langle i,i,i,i\rangle$	40	$\langle c,c,c,c\rangle$
41	$\langle e,i,c,e\rangle$	42	$\langle e,i,c,i\rangle$	43	$\langle e,i,c,c\rangle$	44	$\langle e,i,c,o\rangle$

3.2. The equivalence of the 4-tuple and the extended 9-intersection

Since $T(a_1,b_1)$ and $T(a_2,b_2)$ have five basic relations and $T(a_1,b_2)$ and $T(a_2,b_1)$ have four, the 4-tuple can, theoretically, discern $5\times4\times4\times5=400$ topological relations between SBBRs. However, only the 44 cases included in Table 3 are possible, and the others are impossible due to the fact that there are geometric constraints among simple regions a_1, a_2, b_1, and b_2, such as $a_1\subseteq a_2$ and $b_1\subseteq b_2$. These constraints cause the basic relations in 4-tuple to be dependent, so that some are impossible.

In terms of the geometric constraints among simple regions a_1, a_2, b_1, and b_2, if one relation in 4-tuple is identified, and then the possible relations of other three pairs of regions will be determined. By analyzing the four relations, 17 geometric conditions can be obtained (Table 4).

By applying these 17 conditions, it is possible to reduce the number of cases determined by the 4-tuple representation.

Theorem 1. *When $T(a_1,b_1)=d$, there are 17 possible 4-tuples between two SBBRs.*

Proof. If $T(a_1,b_1)=d$, in terms of condition 1, then $T(a_1,b_2)=\{d,o,i\}$, $T(a_2,b_1)=\{d,o,c\}$, $T(a_2,b_2)=\{d,o,e,i,c\}$.

Furthermore, if $T(a_1,b_2)=o$, then by applying condition 8 we can conclude that $T(a_2,b_1)=\{d,o,c\}$ and $T(a_2,b_2)=\{o,c\}$; then, by applying conditions 12, 13, 15, and 16, four possible cases are obtained: $\langle d,o,o,o\rangle$, $\langle d,o,c,o\rangle$, $\langle d,o,c,c\rangle$, and $\langle d,o,d,o\rangle$.

Table 4
Geometric constraints among the four relations

No.	Existing relation	Possible relations of other three pairs of regions
1	$T(a_1,b_1)=d$	$T(a_1,b_2)=\{d,o,i\}, T(a_2,b_1)=\{d,o,c\}, T(a_2,b_2)=\{d,o,e,i,c\}$
2	$T(a_1,b_1)=c$	$T(a_1,b_2)=\{o,i,c\}, T(a_2,b_1)=c, T(a_2,b_2)=\{o,e,i,c\}$
3	$T(a_1,b_1)=e$	$T(a_1,b_2)=\{o,i,c\}, T(a_2,b_1)=c, T(a_2,b_2)=\{o,e,i,c\}$
4	$T(a_1,b_1)=i$	$T(a_1,b_2)=i, T(a_2,b_1)=\{o,i,c\}, T(a_2,b_2)=\{o,e,i,c\}$
5	$T(a_1,b_1)=o$	$T(a_1,b_2)=\{o,i\}, T(a_2,b_1)=\{o,c\}, T(a_2,b_2)=\{o,e,i,c\}$
6	$T(a_1,b_2)=c$	$T(a_1,b_1)=c, T(a_2,b_1)=c, T(a_2,b_2)=c$
7	$T(a_1,b_2)=d$	$T(a_1,b_1)=d, T(a_2,b_1)=\{d,o,c\}, T(a_2,b_2)=\{d,o,c\}$
8	$T(a_1,b_2)=o$	$T(a_1,b_1)=\{d,o,c\}, T(a_2,b_1)=\{d,o,c\}, T(a_2,b_2)=\{o,c\}$
9	$T(a_1,b_2)=i$	$T(a_1,b_1)=\{d,o,c,i,e\}, T(a_2,b_1)=\{d,o,c,i\}, T(a_2,b_2)=\{o,c,i,e\}$
10	$T(a_2,b_1)=i$	$T(a_1,b_1)=i, T(a_1,b_2)=i, T(a_2,b_2)=i$
11	$T(a_2,b_1)=d$	$T(a_1,b_1)=d, T(a_1,b_2)=\{d,o,i\}, T(a_2,b_2)=\{d,o,i\}$
12	$T(a_2,b_1)=c$	$T(a_1,b_1)=\{d,o,c,i,e\}, T(a_1,b_2)=\{d,o,c,i\}, T(a_2,b_2)=\{o,c,i,e\}$
13	$T(a_2,b_1)=o$	$T(a_1,b_1)=\{d,o,i\}, T(a_1,b_2)=\{d,o,i\}, T(a_2,b_2)=\{o,i\}$
14	$T(a_2,b_2)=d$	$T(a_1,b_1)=d, T(a_1,b_2)=d, T(a_2,b_1)=d$
15	$T(a_2,b_2)=c$ or $T(a_2,b_2)=e$	$T(a_1,b_1)=\{d,o,c,i,e\}, T(a_1,b_2)=\{d,o,c,i\}, T(a_2,b_1)=c$
16	$T(a_2,b_2)=i$	$T(a_1,b_1)=\{d,o,c,i,e\}, T(a_1,b_2)=i, T(a_2,b_1)=\{d,o,c,i\}$
17	$T(a_2,b_2)=o$	$T(a_1,b_1)=\{d,o,c,i,e\}, T(a_1,b_2)=\{d,o,i\}, T(a_2,b_1)=\{d,o,c\}$

If $T(a_1,b_2)=d$, then by applying condition 7 we can conclude that $T(a_2,b_1)=\{d,o,c\}$ and $T(a_2,b_2)=\{d,o,c\}$; then, by applying conditions 11, 12, 13, 14 and 17, five cases are obtained: $\langle d,d,d,d\rangle$, $\langle d,d,d,o\rangle$, $\langle d,d,o,o\rangle$, $\langle d,d,c,o\rangle$, and $\langle d,d,c,c\rangle$.

If $T(a_1,b_2)=i$, then by applying condition 9 we can conclude that $T(a_2,b_1)=\{d,o,c,i\}$ and $T(a_2,b_2)=\{o,c,i,e\}$; in addition, when $T(a_1,b_1)=d$, $T(a_2,b_1)=\{d,o,c\}$ holds; therefore, $T(a_2,b_1)=\{d,o,c,i\}\cap\{d,o,c\}=\{d,o,c\}$. By applying conditions 11, 12, 13, 15, 16, and 17, eight cases are obtained: $\langle d,i,d,o\rangle$, $\langle d,i,d,i\rangle$, $\langle d,i,o,o\rangle$, $\langle d,i,o,i\rangle$, $\langle d,i,c,i\rangle$, $\langle d,i,c,o\rangle$, $\langle d,i,c,c\rangle$, and $\langle d,i,c,e\rangle$.

Therefore, when $T(a_1,b_1)=d$ holds, there are 17 possible cases, each of which is equal to the corresponding one in Table 3.

In the same way, when $T(a_1,b_1)=o$, $T(a_1,b_1)=i$, $T(a_1,b_1)=c$, and $T(a_1,b_1)=e$, then there are, respectively, 9, 7, 7, and 4 possible 4-tuples. Therefore, 44 possible cases, determined by the four relations, are equal to those in Table 3. This indicates that the 4-tuple and the extended 9-intersection determine the same number and the same types of topological relations. Therefore, they are equivalent for the topological relations between SBBRs. □

4. The reasoning of topological relations between SBBRs

In general, reasoning with topological relations can be represented as a composition table. The eight basic topological relations between crisp regions result in an 8×8 table with which their compositions can be computed [16]. For each pair of topological relations between crisp regions, the results of the composition can be found in the table. For example, if *a disjoint b* and *b overlap c*, then the possible topological relations between *a* and *c* are *disjoint*, *meet*, *inside*, *coveredBy*, and *overlap*.

The composition of topological relations between SBBRs is to infer unknown relations between *A* and *C* from existing ones between *A* and *B*, and *B* and *C*. The basic philosophy behind computation of the composition is:

(1) Obtain the 4-tuple representations of existing topological relations between *A* and *B*, and *B* and *C*.
(2) Derive the possible four relations between *A* and *C* from existing 4-tuple representation by applying the composition table for the reduced set of five relations.
(3) Generate all 4-tuple representations of the new topological relations between *A* and *C* using the Cartesian product of the possible four relations.
(4) Remove impossible cases by checking whether a 4-tuple is included in Table 3, and remaining cases are possible topological relations between SBBRs.

A 4-tuple of a topological relation can be found in Table 3 by using a corresponding extended 9-intersection matrix, so step (1) is trivial. The composition table for the reduced set of five relations can be derived by using the composition table for crisp regions. Due to the fact that the four relations are different from those between crisp regions determined by the 9-intersection, a conversion from the former to the latter is needed to compute composition table for the reduced set of relations. For example, let $a_1 contain' b_1$ and $b_1 inside' c_1$, then, to derive possible topological relations between a_1 and c_1 from those between a_1 and b_1, and b_1 and c_1, it is necessary to replace *contain′* with *contain* and *cover*, and *inside′* with *inside* and *coveredBy*, as shown in Table 2. Therefore, the computation will be executed as:

$$
\begin{aligned}
& cover' \circ inside' \\
&\quad = \{contain, cover\} \circ \{inside, coveredBy\} \\
&\quad = \{contain \circ inside\} \cup (contain \circ coveredBy\} \cup \{cover \circ inside\} \cup \{cover \circ coveredBy\} \\
&\quad = \{equal, inside, coveredBy, contain, cover, overlap\} \\
&\qquad \cup (contain, cover, overlap\} \cup \{inside, coveredBy, overlap\} \\
&\qquad \cup \{equal, coveredBy, cover, overlap\} \\
&\quad = \{equal, inside, coveredBy, contain, cover, overlap\} \\
&\quad = \{equal', inside', cover', equal'\}
\end{aligned}
$$

Table 5
The composition table for inner–inner topological relations

	$T(b_1,c_1) = disjoint'$	$T(b_1,c_1) = overlap'$	$T(b_1,c_1) = equal'$	$T(b_1,c_1) = inside'$	$T(b_1,c_1) = contain'$
$T(a_1,b_1) = disjoint'$	$\{d,o,e,i,\mathrm{c}\}$	$\{d,o,i\}$	$\{\mathrm{d}\}$	$\{d,o,i\}$	$\{\mathrm{d}\}$
$T(a_1,b_1) = overlap'$	$\{d,o,c\}$	$\{d,o,e,i,c\}$	$\{\mathrm{o}\}$	$\{o,\mathrm{i}\}$	$\{d,o,c\}$
$T(a_1,b_1) = equal'$	$\{d\}$	$\{o\}$	$\{e\}$	$\{i\}$	$\{c\}$
$T(a_1,b_1) = inside'$	$\{d\}$	$\{d,o,i\}$	$\{i\}$	$\{i\}$	$\{d,o,e,i,c\}$
$T(a_1,b_1) = contain'$	$\{d,o,c\}$	$\{c,o\}$	$\{c\}$	$\{o,i,c,e\}$	$\{c\}$

That is, if $a_1 contain' b_1$ and $b_1 inside' c_1$, then the possible topological relations between a_1 and c_1 are *overlap′*, *inside′*, *cover′*, and *equal′*. In the same way, other compositions of inner-inner topological relations can be computed, as illustrated in Table 5. A set of inner-inner topological relations between a_1 and c_1 can be derived from the inner-inner ones between a_1 and b_1, and b_1 and c_1, and another set from the ones between a_1 and b_2, and b_2 and c_1. Therefore, the result of inner-inner topological relations is the intersection of the two sets. The composition table for inner-outer, outer-inner, and outer-outer topological relations can be computed in the same way as for the inner-inner table.

Definition 2. The 4-tuple of topological relations between SBBRs A and B is $\langle T(a_1,a_1), T(a_1,b_2), T(a_2,b_1), T(a_2,b_2)\rangle$, the one between B and $C\langle T(b_1,c_1), T(b_1,c_2), T(b_2,c_1), T(b_2,c_2)\rangle$. Moreover, the possible topological relations between inner and outer regions of A and C are:

$$T(a_1,c_1) = T(a_1,b_1) \circ T(b_1,c_1) \cap T(a_1,b_2) \circ T(b_2,c_1),$$
$$T(a_1,c_2) = T(a_1,b_1) \circ T(b_1,c_2) \cap T(a_1,b_2) \circ T(b_2,c_2),$$
$$T(a_2,c_1) = T(a_2,b_1) \circ T(b_1,c_1) \cap T(a_2,b_2) \circ T(b_2,c_1),$$
$$T(a_2,c_2) = T(a_2,b_1) \circ T(b_1,c_2) \cap T(a_2,b_2) \circ T(b_2,c_2).$$

Hence, the possible topological relations between A and C are all valid 4-tuples consisting of the Cartesian product of $T(a_1,c_1)$, $T(a_1,c_2)$, $T(a_2,c_1)$ and $T(a_2,c_2)$, denoted by $T(a_1, c_1) \times T(a_1,c_2) \times T(a_2,c_1) \times T(a_2,c_2)$.

Remark 1. Since $T(a_1,c_1)$, $T(a_1,c_2)$, $T(a_2,c_1)$, and $T(a_2,c_2)$ contain all possible relations between the inner and the outer regions of A and C, the 4-tuples combined by the Cartesian product of the four relations are cases between A and C. The results of the Cartesian product, however, may contain some impossible cases that cannot meet the geometric conditions. By applying geometric condition 1–17, we know that among the 400 cases only 44 are realizable. Therefore, those 4-tuples that cannot meet the 17 geometric conditions should be ruled out. To perform this task, we only need to check whether the 4-tuples generated by the Cartesian product are included in Table 3. Only those cases included in Table 3 are the realizable topological relations between A and C.

In Fig. 4a, $T(A,B) = \langle d,d,d,d\rangle$, $T(B,C) = \langle o,o,o,o\rangle$, and $T(a_1,c_1) = T(a_1,c_2) = T(a_2,c_1) = T(a_2,\ c_2) = \{d,o,i\}$, then $\{d,o,i\} \times \{d,o,i\} \times \{d,o,i\} \times \{d,o,i\}$ includes 81 possible 4-tuples, however according to Table 3, only 15 are possible; that is, the 4-tuples corresponding to No. 1, 2, 3, 4, 5, 6, 9, 10, 11, 18, 19, 20, 27, 28, and 39 in Fig. 2 are possible topological relations between A and C.

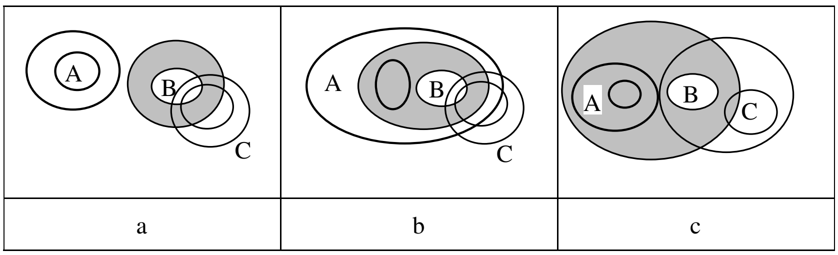

Fig. 4. Reasoning of topological relations between SBBRs.

In Fig. 4b, $T(A,B)=\langle d,i,c,c\rangle$, $T(B,C)=\langle o,o,o,o\rangle$, and $T(a_1,c_1)=T(a_1,c_2)=\{d,o,i\}$, $T(a_2,c_1)=T(a_2,c_2)=\{c,o\}$. Accordingly, there are 36 4-tuples, but only 18 are contained in Table 3. The topological relations corresponding to these 4-tuples are: 6, 7, 8, 9, 10, 12, 13, 14, 15, 18, 19, 21, 22, 23, 25, 28, 29, and 31.

In Fig. 4c, $T(A,B)=\langle d,i,d,i\rangle$, $T(B,C)=\langle d,i,o,o\rangle$, and $T(a_1,c_1)=\{d,o,i\}$, $T(a_1,c_2)=\{d,o,i\}$, $T(a_2,c_1)=T(a_2,c_2)=\{d,o,i\}$. There are 81possible 4-tuples, but 66 are ruled out as they are not contained in Table 3. The remaining 15 cases represent possible topological relations between A and C: 1, 2, 3, 4, 5, 6, 9, 10, 11, 18, 19, 20, 27, 28, and 39.

By comparing the derived topological relations with those in Table 3 and Fig. 2, geometric interpretation and extended 9-intersection matrices of the composition of topological relations can be obtained. A 44×44 composition table for topological relations between SBBRs will be computed in the same way as shown in these examples. (The table can be downloaded at site: http://www.argis.cn/composition_table.pdf.)

5. The reasoning of hierarchical topological relations between SBBRs

In order to satisfy the requirements of various users, Clementini et al. grouped the 44 topological relations between SBBRs into three levels to support the query of uncertain data [8]. The high-level operators were *disjoint*, *touch*, *in*, and *overlap*; the intermediate level comprised 11 operators; and all 44 topological relations constituted the bottom level. When integrating the three levels of operators into query languages, a mechanism is needed to check whether the query sentence is realizable; that is, whether there are some objects in physical world meeting the topological relations in query sentences. When the geometric information is not available, the reasoning of intermediate or top operator can be used to perform this task.

Since the intermediate and top operators are clusters of the 44 bottom operators, their reasoning can be computed by combining the reasoning of the bottom-level operators. Let R_i (for intermediate operators, $1\leqslant i\leqslant 11$; for top ones, $1\leqslant i\leqslant 4$) and R_j (low index $j$has the same domain as i) represent the intermediate or top operators, respectively. If R_i^l ($1\leqslant l\leqslant |R_i|$) is a bottom relation in cluster R_i and R_j^k ($1\leqslant k\leqslant |R_j|$) is one in cluster R_j, then the reasoning of intermediate or top operators can be computed by using Eq. (3).

$$R_i\circ R_j = UpScaling\left(\bigcup_{l=1}^{|R_i|}\bigcup_{k=1}^{|R_j|}R_i^l\circ R_j^k\right) \tag{3}$$

In Eq. (3), $|R_i|$ denotes the number of bottom relations in cluster R_i, so does $|R_j|$. The function *UpScaling* groups a set of bottom operators into intermediate or top clusters according to Table 6.

Eq. (3) can be implemented by the following steps:

(1) According to Table 6, obtaining the set $T_i(A,B)$ of bottom operators in R_i, and $T_j(B,C)$ from R_j;
(2) $k=1$, $l=1$, $T(A,C)=\emptyset$;

Table 6
The hierarchy of topological operators [8]

Top level	Middle level	Bottom level
Disjoint	disjoint (d)	1
Touch	nearlyMeet (nM)	2, 3, 6, 9
	coveredByBoundary (cBB)	4, 5, 10, 11
	coversWithBoundary (cWB)	7, 8, 12, 13
	boundaryOverlap (bO)	14, 15, 16, 17
Overlap	nearlyOverlap (nO)	18, 19, 20, 21, 22, 23, 24, 25, 26
	interiorCoveredByInterior (iCBi)	28, 29, 31, 44
	interiorCoversInterior (iCvi)	34, 35, 37, 44
In	nearlyInside (nI)	27, 30, 39
	nearlyContains (nCt)	33, 36, 40
	nearlyEqual (nE)	32, 38, 41, 42, 43

(2.1) For each bottom operator $T_i^k(A,B)$ in $T_i(A,B)$ and $T_j^l(B,C)$ in $T_j(B,C)$, a set of bottom operators, $T_a(A,C)$, can be derived according to the method presented in Section 4.
(2.2) $T(A,C) = T(A,C) \cup T_a(A,C)$;
(2.3) $l = l + 1$; if $l \leqslant |T_j(B,C)|$, then go to step (2.1); otherwise, go to step (2.4);
(2.4) $k = k + 1$; if $k \leqslant |T_i(A,B)|$, go to step (2.1); otherwise, go to step (3);
(3) $a = 1$, $R(A,C) = \emptyset$;
(3.1) For each bottom operator in $T(A,C)$, an intermediate or top operator $R_a(A,C)$ can be found in Table 6.
(3.2) if $R_a(A,C) \notin R(A,C)$, then $R(A,C) = R(A,C) \cup R_a\,(A,C)$;
(3.3) $a = a + 1$; if $a \leqslant |T(A,C)|$, go to step (3.1); otherwise,go to step (4);
(4) Output the set $R(A,C)$, and end.

Eq. (3) can be performed in three steps: step (1) obtains two sets of the bottom operators in cluster R_i and R_j; step (2) derives a set of bottom operators from each pair of bottom operators in R_i and R_j, according to the method in Section 4, and merges all sets derived into a new set; step (3) groups the new set into clusters by using the function *UpScaling*. The reasoning of intermediate operators is provided in Appendix A (see Table A.1).

6. Related work

Cohn and Gotts [10] used the "egg-yolk" model to represent regions with uncertain boundaries. In that model, the egg represented the inner region, and the white the outer region. The egg consists of the yolk and white. The extended region connection calculus (RCC) classified topological relations between SBBRs into 46 cases in terms of the topological relations between yolk and yolk, yolk and white, and white and white [10,23]. The three types of topological relations are modeled by RCC-5 theory, which have five basic relations: DR, PO, PP, PPI, and EQ.

The first difference between 4-tuple and RCC-5 is that the former is based on sets of points and the latter is based on regions. Our goal is to find an equivalent model with the extended 9-intersection and then relate the 44 relations to the 4-tuples. Thus, we consider the definitions of the interior and the broad boundary in the extended 9-intersection when formalizing the four relations in the 4-tuple; that is, the inner region is an open set and the broad boundary is a closed set. In RCC-5, both the yolk and the white are closed sets. In addition, in our model there are five basic relations between inner and inner, and outer and outer regions, while four basic relations between inner and outer, and outer and inner regions, because a closed set and an open set cannot be equal. In RCC-5, the five basic relations always hold for three pairs of regions.

The second difference lies in the semantics of basic relations. The eight basic relations of RCC-8, DC, EQ, PO, TPPI, NTPPI, TPP, NTPP, and EC, are equivalent to the eight relations of the 9-intersection, *disjoint*, *meet*, *overlap*, *cover*, *contain*, *coveredBy*, *inside*, and *equal*, respectively. The correspondences between RCC-5 and RCC-8 are listed in Table 7, which shows that the semantics of the five relations between yolk and yolk, yolk and white, and white and white are equal. By contrast, the correspondences among the eight basic relations of the 9-intersection and the five or four basic ones of 4-tuples are different, as shown in Table 2. For example, if two regions are *disjoint* or *meet* each other, they are named as relation DR by RCC-5, whereas in 4-tuple the *disjoint* and *meet* relation between a_1 and b_1, a_1 and b_2, and b_2 and a_1 is grouped into *disjoint′*, and the relation *disjoint′* between a_2 and b_2 only refers to *disjoint* of the 9-intersection. The differences between RCC-5 and 4-tuple with respect to the semantics of the basic relations are indicated in Tables 2 and 7. Furthermore, the relation *equal′* does not hold for a_1 and b_2 and a_2 and b_1 while it holds for a_1 and b_1, and a_2 and b_2. In RCC-5, EQ always holds for any pair of regions.

The third difference is that the four relations in 4-tuple can model topological relations between objects with different dimensions, while RCC-5 only the relations between regions. The 4-tuple can be extended to

Table 7
The RCC-8 and RCC-5 sets of basic relations between crisp regions

RCC-5	PO	PPI	PP	DR	EQ
RCC-8	PO	TPPI, NTPPI	TPP, NTPP	EC, DC	EQ

represent topological relations between uncertain regions and uncertain lines, uncertain regions and crisp lines, uncertain regions and crisp points, while RCC can not.

7. Conclusions

In this study, computation methods for inferring new topological relations from the ones between SBBRs are investigated. A 4-tuple is used to represent the topological relations between BBRs. The 4-tuple has the same ability as the extended 9-intersection to discern topological relations. Moreover, the 4-tuple and the extended 9-intersection are interconvertible. The 4-tuple can represent 44 topological relations between SBBRs as the combination of five basic topological relations between crisp regions. This aids in transforming the reasoning of topological relations between SBBRs into the Cartesian product of the reasoning of topological relations between crisp regions.

Future work will focus on evaluating the consistency of BBRs in multi-resolution databases by integrating direction, topology, and qualitative shape description.

Acknowledgement

The work described in this paper was substantially supported by grants from National Natural Science Foundation of China (No. 40271090), and also funded by the Research Fund of LREIS, CAS (No. A0604). We are grateful to the referees of this paper whose comments and suggestions have led to substantial improvements.

Appendix A

See Table A.1.

Table A.1
Composition table for intermediate topological operators

	d	nM	cBB	cWB	bO	nO	iCBi	iCvi	nI	nCt	nE
d	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	d, nM, cBB, cWB, bO, nO, nI, iCBi, iCvi, nE	d, nM, cBB, cWB, bO, nO, nI, iCBi, iCvi, nE	d, nM, cBB	d, nM, cBB	d, nM, cBB, nO, nI, iCBi	d, nM, cBB, nO, nI, iCBi	d, nM, cBB	d, nM, cBB, nO, nI, iCBi	d	d, nM, cBB
nM	d, nM, cBB, cWB, bO, nO, iCBi, nCt, iCvi, nE	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	d, nM, cBB, cWB, bO, nO, nI, iCBi, iCvi, nE	d, nM, cBB, cWB, bO, nO, nCt, iCvi, nE	d, nM, cBB, cWB, bO, nO, iCvi	d, nM, cBB, nO, nI, iCBi, cWB, bO, nE	d, nM, cBB, nO, nI, iCBi	d, nM, cBB, cWB, bO	nM, cBB, nO, nI, iCBi	d, nM, cWB	nM, cBB, d, nO, cWB
cBB	d, nM, cWB	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE	d, nM, cBB, cWB, bO, nO, nI, iCBi	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	d, nM, cBB, cWB, bO, nO, nI, iCBi, iCvi, nE	d, nM, cBB, nO, nI, iCBi, cWB, bO, nE	d, nM, cBB, nO, nI, iCBi	d, nM, cBB, cWB, bO	cBB, nO, nI, iCBi	d, nM, cBB, cWB, bO	cBB, nO, iCBi, d, nM, nI, bO
cWB	d, nM, cBB, cWB, bO, nO, iCBi, nCt, iCvi, nE	d, nM, cBB, cWB, bO, nO, iCBi, nCt, iCvi, nE	nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	d, nM, cBB, cWB, bO, nO, nCt, iCvi	nM, cBB, cWB, bO, nO, nCt, iCvi, nE	nM, cWB, cBB, bO, nO, iCBi, nI, nE	nM, cWB, cBB, bO, nO, nI, iCBi, nE	cWB, bO	nM, cWB, cBB, bO, nO, nI, iCBi, nE	cWB	nM, cWB, bO, nO

Table A.1 (*continued*)

	d	nM	cBB	cWB	bO	nO	iCBi	iCvi	nI	nCt	nE
bO	d, nM, cWB	d, nM, cBB, cWB, bO, nO, iCBi	nM, cBB, cWB, bO, nO, nI, iCBi, nE	d, nM, cBB, cWB, bO, nO, iCBi, nCt, iCvi, nE	nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	nM, cWB, cBB, bO, nO, iCBi, nI, nE	nM, cWB, cBB, bO, nO, nI, iCBi, nE	cWB, bO	cBB, bO, nO, nI, iCBi, nE	cWB,bO	cBB, bO, nO, iCBi, cWB, nI, nE
nO	d, nM, cWB, nO, nCt, iCvi	d, nM, cWB, nO, nCt, iCvi, cBB, bO, nE	nM, cBB, cWB, bO, nO, iCvi, nCt, nE	d, nM, cWB, nO, nCt, iCvi, cBB, bO, nE	nM, cBB, cWB, bO, nO, iCvi, nCt, nE	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	nO, nI, iCBi, nE	d, nM, cBB, cWB, bO, nO, nCt, iCvi, nE	nO, nI, iCBi, nE	d, nM, cWB, nO, nCt, iCvi, cBB, bO, nE	nO, nM, cWB, iCvi, iCBi, cBB, bO, nI, nCt, nE
iCBi	d, nM, cWB	d, nM, cWB, cBB, bO	cBB, bO	d, nM, cWB, cBB, bO	cBB, bO	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE	nI, iCBi, nE, iCvi	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	nI, iCBi, nE	d, nM, cBB, cWB, bO, nO, iCBi, nCt, iCvi, nE	iCBi, cBB, bO, nO, iCvi, nI, nCt, nE
iCvi	d, nM, cWB, nO, nCt, iCvi	d, nM, cWB, nO, nCt, iCvi	nM, cBB, cWB, bO, nO, nCt, iCvi, nE	d, nM, cWB, nO, nCt, iCvi	nM, cBB, cWB, bO, nO, nCt, iCvi, nE	nO, nCt, iCvi, nE	nO, nI, iCBi, nE, nCt, iCvi	nCt, iCvi, nE, iCBi	nO, nI, iCBi, iCvi, nE	nCt, iCvi, nE	nO, iCvi, nCt, iCBi, nI, nE
nI	d	d, nM, cBB	cBB	d, nM, cWB, cBB, bO	cBB, bO	d, nM, cBB, nO, nI, iCBi, cWB, bO, nE	nI, iCBi, nE	d, nM, cBB, cWB, bO, nO, nI, iCBi, iCvi, nE	nI	d, nM, cBB, cWB, bO, nO, nI, iCBi, nE, nCt, iCvi	nI, cBB, bO, nO, iCvi, nE, iCBi
nCt	d, nM, cWB, nO, nCt, iCvi	nM, cWB, nO, nCt, iCvi	nM, cBB, cWB, bO, nO, nCt, iCvi, nE	cWB, nO, nCt, iCvi	cWB, bO, nO, nCt, iCvi, nE	nO, nCt, iCvi, nE	nO, iCBi, nCt, iCvi, nE	nCt, iCvi, nE	nO, nI, iCBi, nE, nCt, iCvi	nCt	nO, nCt, iCvi, nE, iCBi
nE	d, nM, cWB	nM, cBB, nO, d, cWB	cBB, nM, nO, bO	cWB, bO, nO, iCvi, nCt, d, nM	bO, cWB, nO, iCvi, nCt, nE, cBB	nM, cBB, nO, iCBi, nI, cWB, bO, nE, iCvi, nCt	iCBi, nO, iCvi, nCt, nE, nI	cWB, bO, nO, iCBi, iCvi, nI, nE, nCt	nI, nO, iCvi, nE, iCBi	cWB, bO, nO, iCBi, nCt, nE, iCvi	nE, bO, nO, nI, iCBi, iCvi, nCt

References

[1] A.I. Abdelmoty, B.A. El-Geresy, A general method for spatial reasoning in spatial databases, in: N. Pissinou, A. Silberschatz, E.K. Park, K. Makki (Eds.), International Conference on Information and Knowledge Management, ACM Press, Baltimore, 1995, pp. 312–318.

[2] A.I. Abdelmoty, B.A. El-Geresy, A general approach to the representation of spatial relationships, Technical Report, Department of Computer Studies, University of Glamorgan, 1995.

[3] A.I. Abdelmoty, C.B. Jones, Towards maintaining consistency of spatial databases, in: Sixth International Conference on Information and Knowledge Management, Las Vegas, Nevada, United States, 1997, pp. 293–300.

[4] E. Clementini, P. Di Felice, P.A. Oosterom, Small set of formal topological relationships suitable for end-user interaction, in: The 3rd International Symposium on Advances in Spatial Databases, Lecture Notes in Computer Science, vol. 692, 1993, pp. 277–295.
[5] E. Clementini, P. Di Felice, An algebraic model for spatial objects with indeterminate boundaries, in: P. Burrough, A. Frank (Eds.), Geographic Objects with Indeterminate Boundaries, Taylor & Francis, London, 1996, pp. 155–169.
[6] E. Clementini, P. Di Felice, Approximate topological relations, International Journal of Approximate Reasoning 16 (2) (1997) 173–204.
[7] E. Clementini, P. Di Felice, K. Koperski, Mining multiple-level spatial association rules for objects with a broad boundary, Data & Knowledge Engineering 34 (3) (2000) 251–270.
[8] E. Clementini, P. Di Felice, A spatial model for complex objects with a broad boundary supporting queries on uncertain data, Data & Knowledge Engineering 37 (3) (2001) 285–305.
[9] J. Chen, C.M. Li, Z.L. Li, G.A. Christopher, Voronoi-based 9-intersection model for spatial relations, International Journal Geographical Information Science 15 (3) (2001) 201–220.
[10] A.G. Cohn, N.M. Gotts, The 'egg-yolk' representation of regions with indeterminate boundaries, in: P.A. Burrough, A.U. Frank (Eds.), Geographic Objects with Indeterminate Boundaries, Taylor & Francis, London, 1996, pp. 171–187.
[11] M.J. Egenhofer, R. Franzosa, Point-set topological spatial relations, International Journal of Geographical Information Systems 5 (2) (1991) 161–174.
[12] M.J. Egenhofer, J. Herring, Categorizing binary topological relations between regions, lines and points in geographic databases, Technical Report, Department of Surveying Engineering, University of Maine, 1991.
[13] M.J. Egenhofer, K. Al-Taha, Reasoning about gradual changes of topological relationships, in: A. Frank, I. Campari, U. Formentini (Eds.), Theory and Methods of Spatio-Temporal Reasoning in Geographic Space, Lecture Notes in Computer Science, vol. 639, 1992, pp. 196–219.
[14] M.J. Egenhofer, J. Sharma, Topological consistency, in: D.J. Cowen (Eds.), Fifth International Symposium on Spatial Data Handling, Charleston, SC, 1992, pp. 335–343.
[15] M.J. Egenhofer, J. Sharma, Assessing the consistency of complete and incomplete topological information, Geographical Systems 1 (1) (1993) 47–68.
[16] M.J. Egenhofer, Deriving the composition of binary topological relations, Journal of Visual Languages and Computing 5 (2) (1994) 133–149.
[17] M.J. Egenhofer, E. Clementini, and P. Di Felice, Evaluating inconsistencies among multiple representations, in: Sixth International Symposium on Spatial Data Handling, Edinburgh, Scotland, 1994, pp. 901–920.
[18] M.J. Egenhofer, Query processing in Spatial-Query-by-Sketch, Journal of Visual Languages and Computing 8 (4) (1997) 403–424.
[19] M.D. Hendricks, Structuring a wayfinder's dynamic and uncertain environment, Ph.D. Thesis, University of Maine, 2004.
[20] J.A.C. Paiva, Topological equivalence and similarity in multi-representation geographic databases, Ph.D. Dissertation, Department of Surveying Engineering, University of Maine, 1998.
[21] A. Rodríguez, Inconsistency issues in spatial databases, in: L. Bertossi, T. Hunter, T. Schaub (Eds.), Inconsistency Tolerance, Lecture Notes in Computer Science, vol. 3300, 2005, pp. 237–269.
[22] A. Stefanidis, P. Agouris, C. Georgiadis, M. Bertolotto, J.D. Carswell, Scale- and orientation-invariant scene similarity metrics for image queries, International Journal of Geographical Information Science 16 (8) (2002) 749–772.
[23] J.G. Stell, Part and complement: fundamental concepts in spatial relations, Annals of Mathematics and Artificial Intelligence 41 (1) (2004) 1–17.
[24] N. Tryfona, M.J. Egenhofer, Consistency among parts and aggregates: a computational model, Transactions in GIS 1 (3) (1997) 189–206.
[25] M. Worboys, E. Clementini, Integration of imperfect spatial information, Journal of Visual Languages and Computing 12 (1) (2001) 61–80.
[26] S.S.T. Yau, Q.L. Zhang, On completeness of reasoning about planar spatial relationships in pictorial retrieval systems, Communications in Information and Systems 4 (3) (2004) 211–234.